Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Lecture: 11

Linear Time Invariant Systems (LTI)

* LTI system can be configured as

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$$

LTI system should not vary the time to be really time-invariant system

$$y(t-t_0)=H\{x(t-t_0)\}$$

where H is the operation of the system

thus, in other words, if $x(t)$ has a time delay $(t_0) \longrightarrow x(t-t_0)$, the output should produce the same time relation, $y(t-t_0)$

* $\searrow$

* In fact, $y(t)$ can be produced mathematically as follows

$$\boxed{y(t) = h(t) * x(t)}$$

where $h(t)$ is the function of the system, by taking the Fourier transform of both sides, we get

* Hence $\boxed{Y(f) = H(f)X(f)}$

$$\therefore \boxed{H(f) = \frac{Y(f)}{X(f)}}$$

* $H(f)$ is called the transfer function of the system.

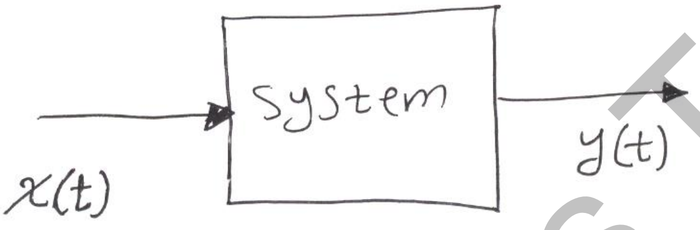
* Note: if $H(f) = 1 \Rightarrow Y(f) = X(f)$ {i/p = o/p}.

OR If $X(f) = 1$ then the output $Y(f)$ is the same as the system function $H(f)$

$\boxed{H(f) = Y(f)}$ this can be achieved if the input $x(t) = \delta(t)$.

Systems:

A system is a mathematical model that represents the transformation of some input signal $x(t)$ into an output signal $y(t)$.



- For instance, a system could be some circuit for example.
- A simple system is a resistor, using Ohm's Law $v(t) = R\, i(t)$, hence, the system relationship is $\underbrace{y(t)}_{o/P} = \underbrace{R}_{system} \underbrace{x(t)}_{i/P}$

- Thus, in general, the mathematical representation for any system [black-box] is

$$\boxed{y(t) = F\{x(t)\}}$$

- For the above resistor's example, the Function F is only a multiplication by R.

- Another Example for a system is the relationship between the current and voltage of a capacitor :-

$$\boxed{i(t) = C\,\frac{dv}{dt}}$$

thus the function of the system in this case is the derivative $\frac{d}{dt}$ multiplied by the constant C.

- A system is called continuous-time if $x(t)$ & $y(t)$ are continuous time signals.
- It is possible to have systems with input & output signals that are discrete time signals.

* Some Important Properties of Systems:

① Memoryless Systems

if the output $y(t)$ depends on the input $x(t)$ at the same time, then the system is called memoryless

$$\boxed{y(t) = a\,x(t)}$$ a is constant

② System With Memory :-

a memory system's output depends on the values of the input at previous times.

* For example: adding up the values of the input signal from all past times up to the present time (integrate),

$$\boxed{y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau}$$

EX. Determine if the following systems are memoryless:

ⓐ $y(t) = \sin(t)\cos(t)$, ⓑ $y(t) = \int_{-\infty}^{t/3} x(t)\, d\tau$,

ⓒ $y(t) = \int_{t_0}^{t} x(\tau)\, d\tau$, consider $x(t) = t\, e^{-t}$.

Solution ⓐ memoryless because $y(t)$ depends on the present value of $x(t)$.

ⓑ has a memory, because it depends on the previous values of $x(t)$.

ⓒ integrate by parts $y(t) = e^{-t_0}(1+t_0) - e^{-t}(1+t)$, hence the system has memory because it depends on the previous values of $x(t)$.

3 Causal & Noncausal Systems :

* Causal system's output depends only on the present or earlier times of the input

* OR : The causal system does not inticipate future values of the input.

* ALL memoryless systems are causal.

* NonCausal systems : Anticipte the future values of the input signal.

$$\boxed{y(t) = c\, x(t+\alpha)}$$ non-causal system

4 Linear Systems :

$$x(t) \longrightarrow \boxed{\begin{array}{c}\text{System}\\ F\{x(t)\}\end{array}} \longrightarrow y(t)$$

if $F\{x_1(t)\} = y_1(t)$ & $F\{x_2(t)\} = y_2(t)$

then $a x_1(t) + b x_2(t) = a y_1(t) + b y_2(t)$

the system is Linear

→ procedure to determine the system is Linear. →

* To Determine the system Linearity, you have to follow certain procedure, & to understand these procedures, lets start with an example.

EX. let $y(t) = R\,x(t)$, R is constant, determine if the system is linear or not linear.

solu. we consider two input & output signals multiplied by scalars.
& since $y(t) = R\,x(t)$,

we have $y_1(t) = R\,x_1(t)$,

$y_2(t) = R\,x_2(t)$,

and $a y_1(t) + b y_2(t) = aR\,x_1(t) + bR\,x_2(t)$

Now we use the sum $a x_1(t) + b\,x_2(t)$ as input to the system

$$F\{a x_1(t) + b\,x_2(t)\} = R[a x_1(t) + b\,x_2(t)]$$
$$= aR\,x_1(t) + bR\,x_2(t)$$

the last expression is equal to $a y_1(t) + b y_2(t)$,

Hence the system is Linear.

EX. 2 Determine if $y(t) = x^3(t)$ linear or nonlinear.

Solution Step ① $a y_1(t) + b y_2(t) = a x_1^3(t) + b x_2^3(t)$

Step ② $F\{a x_1(t) + b x_2(t)\} = [a x_1(t) + b x_2(t)]^3$

$$= a_1^3 x_1^3(t) + 3 a^2 b x_1^2(t)\, x_2(t) + 3 a b^2 x_1(t)\, x_2^2(t) + b^3 x_2(t)$$

But $F\{a x_1(t) + b x_2(t)\} \neq a y_1(t) + b y_2(t)$

Hence, the system is not linear

EX.3 Is $y(t) = \dfrac{d^2 x}{dt^2}$ linear? Explain your answer mathematically.

Solution Step ① we consider two i/p & o/p signals multiplied by scalar

~~$y_1(t) = x_1(t)$~~

$$a y_1(t) + b y_2(t) = a \frac{d^2 x_1}{dt^2} + b \frac{d^2 x_2}{dt^2}$$

Step ② Now we use the sum $a x_1(t) + b x_2(t)$ as input to the system

$$F\{a x_1(t) + b x_2(t)\} = \frac{d^2}{dt^2}[a x_1(t) + b x_2(t)]$$

$$= \frac{d^2}{dt^2}[a x_1(t)] + \frac{d^2}{dt^2}[b x_2(t)]$$

$$= a \frac{d^2 x_1}{dt^2} + b \frac{d^2 x_2}{dt}$$

Hence $F\{a x_1(t) + b x_2(t)\} = a y_1(t) + b y_2(t)$

Thus, the system is linear.

⑤ Time-Invariance Systems

~~if and~~ If $y(t) = F\{x(t)\}$

then the system is time invariant or Linear time invariant

if & only if

$$y(t \mp C) = F\{x(t \mp C)\}$$

⑥ System Stability

if i/p $x(t)$ is bounded, i.e. $|x(t)| \leqslant \alpha$

and o/p $y(t)$ is bounded, i.e. $|y(t)| \leqslant \beta$

where α & β are finite constants then

the system is Bounded Input Bounded Output

(BIBO) stable

EX. Determine if $y(t) = \sin[x(t)]$ is memoryless, causal, stable & time-invariant system.

solu. Given $y(t) = \sin[x(t)]$

① the system is memory less since the output depends only on the i/p at the present time.

② A memoryless system is causal, thus $y(t)$ is causal because $y(t)$ does not anticipate any future values of the input and depends only on the present time.

③ Stability: $y(t) = \sin[x(t)]$ is stable because we know that $|\sin(\tau)| \leq 1$ for all values of τ and so the system is bounded.

④ Time-Invariancy:

First we time-shif the input to give $x(t-\tau)$.

$y_\tau(t) = \sin[x(t-\tau)]$ —— (1)

$y(t-\tau) = \sin[x(t-\tau)]$ —— (2)

similar ∴ So the system is time-invariant.

<u>H.W.</u>: check the following systems and determine their memorylessness, Causality, Stability, and time-invariancy.

① $y(t) = 2e^{-4\cos[x(t)]}$ for all $t \geq 0$

② $y(t) = x(t-6)$

③ $y(t) = x^2(t)$

④ $y(t) = x(t)\cos t$

⑤ $y(t) = \frac{dx}{dt}$

⑥ $y(t) = \int_{-\infty}^{t/3} x(s)\, ds$

<u>Ans</u>

① memoryless, causal, bounded stable, time-invariant

② has memory, causal, bounded stable time-invariant

③ memoryless, causal, bounded stable, time-invariant.

④ memoryless, causal, not stable, not time-invariant.

⑤ not memoryless, causal, not stable, time-invariant.

⑥ not memoryless (has memory), causal, not stable, not time invariant.